

## PRE-I.I.T. ENTRANCE TEST

Max: Marks 100

Time: 3 hours

## SECTION A

1. A uniform rod of mass  $M$  and length  $L$  lies along the  $x$ -axis with centre at the origin. Find the total gravitational field at a point on the axis at  $x_0$  ( $x_0 > \frac{L}{2}$ ) [4]
2. A straight, smooth tunnel is dug through a spherical planet whose density  $\rho$  is constant. The tunnel passes through the centre of the planet and is perpendicular to the planet's axis of rotation. The planet rotates with an angular velocity  $\omega$  so that objects in the tunnel have no acceleration relative to the tunnel. Find the relation between  $\rho$  and  $\omega$  for this to be valid. [4]
3. A gun is mounted on a gun-carriage, movable on a smooth horizontal plane; a shot is fired and leaves the gun in a direction inclined at an angle  $\theta$  to the horizontal. If the mass of the gun and its carriage be  $n$  times that of the shot, show that  $\tan \theta = (1 + 1/n) \tan \alpha$ , where  $\alpha$  is the angle of elevation of the gun with the horizontal. [5]
4. The intensity level 2 m from a motorcycle is 90 dB. Find the distance at which the intensity is 60 dB. (ignore the absorption of sound). [4]
5. Three component sinusoidal waves have the same period, but their amplitudes are in the ratio 1,  $\frac{1}{2}$  and  $\frac{1}{3}$  and their phase angles are 0,  $\frac{\pi}{2}$  and  $\pi$  respectively. Show that the resultant wave is also a sine wave which has the same period but amplitude  $\frac{5}{6}$  and phase angle  $37^\circ$ . [4]
6. Find the least value of the refracting angle of prism made of glass of refracting index  $\frac{7}{4}$  so that no rays incident on one of the faces containing this angle can emerge from the other. [4]
7. A telescope consists of two glass balls of refractive index 1.5 and of radii 5 cm and 1 cm. If the bigger ball is used as an objective, what is the distance between the centres of the balls. What is the magnifying power? [4]
8. Given the distance of the earth from the Sun is 215 times the radius of Sun, Stefan - Boltzmann constant is  $5.7 \times 10^8 \text{ W m}^{-2} \text{ K}^{-4}$  and power received per metre<sup>2</sup> of earth's surface from the Sun's radiation is  $1340 \text{ W/m}^2$ . Find the surface temperature of the Sun. [5]
9. How many degrees of freedom have the gas molecules if under standard conditions, the gas density is  $1.3 \text{ mg/cm}^3$  and the velocity of sound propagation is  $330 \text{ m/s}$ ? [4]
10. A particle of mass  $m$  is placed in a one-dimensional potential field where the potential energy of the particle depends on the coordinate as  $U(x) = U_0(1 - \cos ax)$ ;  $U_0$  and 'a' are constants. Find the period of small oscillations that the particle performs about the equilibrium position. [5]
11. A network is formed of uniform wire in the shape of a rectangle of sides  $2a, 3a$  with parallel wires arranged so as to divide the internal space into six squares of side 'a', the contact at points of intersection being perfect. Show that if a current enters the