Quadratic Equation and applications

A quadratic equation can be written in the form of

$ax^2+bx+c=0$

Where a,b &c are real numbers, $a \neq 0$. This is called the standard form of quadratic equation

For a general quadratic equation in the form

$ax^2+bx+c=0$, $a \neq 0$

Divide by a, $x2+\frac{b}{a}x+\frac{c}{a}=0$, and substract $\frac{c}{a}$ from both the sides

$x^2+\frac{b}{a}x=-\frac{c}{a}$

*we now want to add a constant term to both sides of the equation, so that the left hand side of the equation is a perfect square, i.e., of the form $(x+d)^2$(*indoing so, we shall have "completed the square"). Since

$$(x+d)^2=x^2+2dx+d^2$$

We must have 2d= b/a , or d= b/2a. Thus the tern we have to add is $d^2 = b^2 / 4a^2$, we obtained

$x^2+\frac{b}{a}x + b^2 / 4a^2 = b^2 / 4a^2 - c/a$

Rewriting the left hand side as a square and putting the right hand side over a common denominator, we obtain

$(x+b/2a)^2 = b^2 - 4ac / 4a^2$

Then (by the preceding special case),

$x+\frac{b}{2a}=\frac{\sqrt{b2-4ac}}{\sqrt{4a2}}$ or

$x+\frac{b}{2a}=\frac{\sqrt{b2-4ac}}{2a}$

$x= -\frac{b}{2a}\pm\frac{\sqrt{b2-4ac}}{2a}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The last formula gives us all the real roots of any quadratic equation which is written in standard form. The formula, called the quadratic formula, is very important.

Quadratic equation

Solve the following equation using quadratic equation

$2x^2+x=6$

We rewrite the equation in standard form, $ax^2+bx+c=0$

$2x^2+x-6=0$

We see a=2,b=1,c=-6. Write the formula and substitute:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2-4(2)(-6)}}{2(2)}$$

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$$= \frac{-1 \pm \sqrt{49}}{4}$$

$$= \frac{-1 \pm 7}{4}$$

Hence

$$x = -1 + \frac{7}{4} = \frac{6}{4} = \frac{3}{2} \text{ or } x = -1 - \frac{7}{4} = -\frac{8}{4} = -2$$

Applications of Quadratic eqation

Calculate the pH of a $1.00\text{x}10^{-2}$M H_2SO_4 solution

Solution: The major species in solution are

H^+, HSO_4^-, H2O

Initial concentration (mol/L)		Equilibrium concentration (mol/L)
$[HSO_4^-]_0 = 0.01.$	x mol/L HSO_4^- dissociates	$[HSO_4^-] = 0.01-x$
$[SO_4^{-2}]_0 = 0$	-->	$[SO_4^{-2}] = x$
$[H^+]_0 = 0.01$	to reach equilibrium	$[H^+] = 0.01-x$

Substituting the equilibrium concentrations into the expression for k_{a2} gives

$1.2 \times 10^{-2} = k_{a2} = [H^+]\ [SO_4^{-2}] / [\ HSO_4^-] = \frac{(0.01+x)(x)}{(0.01-x)}$

Leads to $(1.2 \times 10^{-2})(0.01-x)=(0.01+x)(x)$

$(1.2 \times 10^{-4})-(1.2 \times 10^{-2})x = (1.0 \times 10^{-2})x+(x)^2$

$X^2+(2.2 \times 10^{-2})x-(1.2 \times 10^{-4})=0$

This equation can be solved by using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where a=1, $b=2.2 \times 10^{-2}$ and $c=-(1.2 \times 10^{-4})$using above equative , we get $x=4.5 \times 10^{-3}$

Thus $[H^+]$ =0.010+x=0.010+0.0045=0.0145

pH=1.84