Scientific notation and metric prefixes

Introduction

There are some 3,000,000,000bp making up human genomic DNA in a haploid cell. It will weigh approximately 0.0000000000035gms. To amplify a specific segment of DNA using the polymerase chain reaction(PCR), 0.00000000001 moles of each of two primers should be added to a reaction that can produce, following some number of cycles of the PCR, over 1,000,000,000 copies of the target gene.

Molecular biologist work with extremes of numbers. Two shorthand methods have been adopted that bring both enormous and infiniticimal quantities back into the realm of manageability. These methods uase scientific notations. They require the use of exponents and understanding of significant digits.

Problem 1. How many significant are there in each of the following measurement

A) 3,002,000,000bp(base pairs)

B) 0.00305 grams

c) 0.00220 litres*

*volume delivered with a calibrated micropipettor

Solution:

<u>**Given number**</u>	<u>**No. of significant digits**</u>	<u>**The significant digits are**</u>
A) 3,002,000,000bp	4	3002
B)0.00305 grams	3	305
c) 0.00220 litres*	3	220

Guidelines for rounding of significant digits

1.A number 26.6884 can be rounded as 26.7

2. 7.7221 is rounded off to 7.7

Exponenents & scientific notations

Move the decimal point to the right of the leftmost nonzero digit. Count the number of places the decimal has been moved.

For numbers greater than ten(where the decimal was moved to the left) positive exponent

For numbers less than one(where the decimal was moved to the right) negative exponent

Problem : write the following numbers in scientific notation

(a) 3,002,000,000
(b) 89
(c) $70.53x10^{23}$

Solution

A) 3,002,000,000

Move the decimal to the left nine places so that it is positioned to the right of the leftmost nonzero digit.

3.002000000

Write the new number to include all nonzero significant figures, and drop all zeros outside of these numerals. Multiply the new number by 10, and use a positive 9 as the exponent since the given number is greater than 10 and the decimal was moved to the left nine positions.

$3,002,000,000=3.001X10^{9}$

B) 89

Move the decimal to the left one place so that it is positioned to the right of the leftmost nonzero digit. Multiply the number by 10, and use a positive 1 as an exponent since the given number is greater than 10 and decimal was moved to the left one position.

$89=8.9X10^{1}$

C) $70.53x10^{23}$

Move the decimal to the left one place so that it is positioned to the right of the leftmost nonzero digit. Since the decimal eas moved one position to the left, add 1 to the exponent(23+1=24=new exponent value).

$70.53x10^{23}=7.53X10^{24}$

Write the following numbers in scientific notation:

a) 0.000000000016

b)$547.38X10^{-7}$

Solution

A) 0.000000000016
Move the decimal to the right 11 place so that it is positioned to the right of the leftmost nonzero digit. Write the new number to include all numbers between the left most nonzero digit. Write the new number to include all numbers between the leftmost and rightmost significant (nonzero) figures. Drop all zeros lying outside these numerals. Multiply the number by10 and use a negative 11 as the exponent since original number is less than one and the decimal was moved to the right by 11 places.
$0.000000000016=.1.6X10^{-11}$

b)$547.38X10^{-7}$

Move the decimal point two places to the left so that it is positioned to the right of the left most nonzerodigit.Since the decimal is moved two places to the left, add a positive to the exponent value (-7+2=-5)

$547.38X10^{-7}=5.4738X10^{-5}$

Converting numbers from scientific notation to decimal notation

To change a number expressed in scientific notation to decimal form

1. If the exponent of 10 is positive, move the decimal point to the right the same number of positions as the value of the exponent. If necessary, add zeros to the right of the significant digits to hold positions from the decimal point
2. If the exponent of 10 is negative, move the decimal point to the left the same number of positions as the value of the exponent. If necessary, add zeros to the left of the significant digits to hold positions from the decimal point

Problem: write the following numbers in decimal form

(a)$5.47x10^{5}$

(b)$4.5x10^{-4}$

Solution:

(a)$5.47x10^{5}$

Move the decimal point five places to the right, adding three zeros to hold the decimal's place from its former position

$5.47x10^{5}$=547,000.0

(b)$4.5x10^{-4}$

The decimal point is moved four places to the left. Zeros are added to hold thedecimal point's position.

$4.5x10^{-4}=0.00045$

Adding and substracting numbers written in scientific notation

When Adding or substracting numbers written in scientific notation, it is simplest first convert the numbers in the equation to the same power of ten as that of the highest exponent. The exponent value then does not change when the computationis finally performed.

Problem: perform the following computation

$(9x10^4)+(6x10^4)$

$=15x10^4$ numbers added

$=1.5x10^5$ number rewritten in standard scientific notation form

$=2x10^5$ number rounded off to 1 significant digit

Multiplying and dividing numbers in scientific notation

Exponent laws used in multiplication and division for numbers written in scientific notation includes

Product rule: when multiplying using scientific notations, the exponents will be added

Quotient rule: when dividing scientific notation, the exponent of denominator is substracted from the exponent of the numerator

Problem: Calculate the product

$(6x10^4)x(5x10^3)$

$= (6x5)x(10^4x10^3)$ Use commutative and associative laws to group like terms

$=30x10^7$ Exponents are added.

$=3x10^8$ Numbers written in scientific notation form

Metric prefixes

A metric prefix is short hand notation use to denote very large ore very small values of a basic unit as an alternative to expressing them as powers of 10. Basic units frequently used in the biological sciences include meters, grams,moles and litres. Because of their simplicity metric prefixes have found wide application in molecular biology.

Symbol	Prefix	Multiplication Factor	
E	exa	10^{18}	1,000,000,000,000,000,000
P	peta	10^{15}	1,000,000,000,000,000
T	tera	10^{12}	1,000,000,000,000
G	giga	10^{9}	1,000,000,000
M	mega	10^{6}	1,000,000
k	kilo	10^{3}	1,000
h	hecto	10^{2}	100
da	deka	10^{1}	10
d	deci	10^{-1}	0.1
c	centi	10^{-2}	0.01
m	milli	10^{-3}	0.001
μ	micro	10^{-6}	0.000,001
n	nano	10^{-9}	0.000,000,001
p	pico	10^{-12}	0.000,000,000,001
f	femto	10^{-15}	0.000,000,000,000,001
a	atto	10^{-18}	0.000,000,000,000,000,001

Referring the above table, 1 nanogram is equivalent to $1x10^{-9}$ grams, Therefore $1x10^{9}$ nanograms per gram. Likewise, on (μL) is equivalent to $1x10^{-6}$ litres, there are $1x10^{6}$ μL/L.

Conversion factors and cancelling terms

Translating a measurement expressed with on e metric prefix into a equivalent value expressed using a different metric prefix is called a conversion

$\frac{1x10^{-6}\mu g}{g}$ and $\frac{1g}{1x10^{-6}\mu g}$

This can be used to convert the grams to micrograms or μg to gms. The final metric prefix expression desired should appear in the equation as a numerator value in the conversion factor

Problem

(a) There are approximately $7x10^{9}$ bp per human diploid genome. What is the number expressed as kilobp?
(b) Convert 0.05 μg_ into ng
(c) Covert 0.0035mL into μL

(a) Solution
$7x10^{9}$ bp = nKb

Multiply by a conversion factor relating kb to bp with kb as a numerator

$7x10^9$ bp x $\frac{1 \text{ kb}}{1x10^3 \text{bp}}$ =nkb

Cancel identical terms (bp) appearing as numerator and denominator, leaving kb as a numerator value

$\frac{(7x10^9 \text{ bp}) \ (1 \text{ kb})}{1x10^3 \text{bp}}$ =nkb

The exponent of denominator is substracted from exponent of the numerator

$\frac{7}{1}x\ 10^{9-3}$kb = $7x10^6$ kb =nkb

Therefore, $7x10^9$ bp is equivalent to$7x10^6$ kb

(b) Convert 0.05 µg into ng

Multiply by a conversion factor relating g to µg and ng to g with ng as a numerator.Convert 0.005 µg to its equivalent in scientific notation($5x10^{-2}$) µg

$5x10^{-2}$ µg x $\frac{1 \text{ g}}{1x10^6 \text{ µg}}$ x $\frac{1x10^9 \text{ ng}}{\text{g}}$= n ng

Cancel identical terms appearing as numerator and denominator, leaving ng as a numerator value, multiplying numerator and denominator values, and then group like terms

$\frac{(5x1x1)(10^{-2} \ x \ 10^9)\text{ng}}{(1x1)(10^6)}$ = n ng

Numerator exponents are added

$\frac{5x10^{-2+9}\text{ng}}{1x\ 10^6}$ =$\frac{5x\ 10^7\text{ng}}{1x\ 10^6}$=n ng

The exponent of denominator is substracted from exponent of the numerator

$\frac{5}{1}$ x 10^{7-6}ng=5x 10^1 ng = n ng

Therefore, 0.05 µg is equivalent to5x 10^1 ng

(c) Covert 0.0035mL into µL

Convert 0.0035mL into scientific notation. Multiply by a conversion factor relating L to mL and with µL to L with µL as a numerator m

$3.5x10^{-3}$ mL x $\frac{1 \text{ L}}{1x10^3 \text{ mL}}$ x $\frac{1x10^6 \text{ µL}}{1\text{L}}$ = n µL

Cancel identical terms appearing as numerator and denominator, leaving µL as a numerator value, multiplying numerator and denominator values, and then group like terms

$\frac{(3.5x1x1)(10^{-3} \times 10^{6})\ \mu L}{(1x1)(10^{3})} = n\ \mu L$

Numerator exponents are added

$\frac{3.5x10^{-3+6}\ \mu L}{1x\ 10^{3}} = \frac{3.5x\ 10^{3}\ \mu L}{1x\ 10^{3}} = n\ \mu L$

The exponent of denominator is substracted from exponent of the numerator

$\frac{3.5}{1}\ x\ \ 103^{-3}\ \mu L = 3.5x\ 10^{0}\ \mu L = 3.5\ \mu L = n\ \mu L$

Therefore, 0.0035 mL is equivalent to 3.5 µL

Express in scientific notations

1) $7.13x10^{-4}+6.21x10^{-5}$

$=7.13x10^{-4}+0.621x10^{-4}$

$= (7.13+0.621)\ x10^{-4}$

$=7.75x10^{-4}$

2) $(7x10^{-7})(8x10^{3})$

$=56x10^{-7}x10^{3}$

$=56x10^{-4}$

$=5.6x10^{-3}$

3) $4x10^{4}/8x10^{8}= \frac{4}{8}\ x10^{4-8}$

$=0.5x10^{-4}$

$=5x10^{-5}$

Problem:

The weight of a certain microorganism is $5x10^{-8}$gms. How many organisms are there in a population whose total weight is 0.25gms.

Solution:

The number of microorganisms = $0.25/5x10^{-8}$

$= 0.05x10^8$

$=5x10^6$

Consider a culture of bacteria whose initial weight is 1 gm. Its weight doubles every hour. After one hour the weight will be two grams, after two hours 4gms, after threehours 8gms.

y(0)=1

y(1)=2

$y(1)=4=(2^2)$

$y(3)=8=(2^3)$

$y(n)=(2^n)$

After half an hour $y(1/2)= 2^{1/2} =\sqrt{2} =1.414$

Ater 3/4th hour, $y(3/4)= 2^{3/4}$

=4th root of$\sqrt{8}$

=1.682

After 1 ½ hour

$y(3/2)= 2^{3/2} =\sqrt{8}= 2.828$

we can calculate the weight $y(x)=2^x$ fpor any value of x that is rational number.

The value obtained can be plotted as a graph

$Y=2^x$

x	0	1/4	1/2	3/4	1	5/4	3/2	7/4	2
y	1	1.18	1.414	1.682	2	2.37	2.82	3.36	4

A function of the type $y=a^x$ is called an exponential function. When a> 1 the function is said to be growing exponential function, whereas a<1 it is said to be a decaying exponential function.

The graph of $y=a^x$ when a<1 is shown below

When a<1

a^x decreases as x gets larger and approaches zero, as $x \rightarrow \infty$

graph

Example: A culture of bacteria initially weighs one gram and is doubling in size every hour. How long will it take to reach a weight of 3gms?

After n hours the weight is $y(n)= 2^n$

We need to calculate the value of n for which y(n)=3

$2^n=3$

Take logs on both sides of equation, we obtain nlog2=log3

$$n= \frac{\log 3}{\log 2} = \frac{0.4771}{0.3010} =1.58$$

it takes 1.58hrs for the culture to reach weight of 3 gms

Logarithms

ab=antilog (loga+logb)

Calculate 6.17x1.42

a=6.17;loga=0.7903

b=1.42;logb=0.1523

0.9426

ab= antilog (loga+logb)

ab= antilog (0.9426)

=8.762

Division

a/b=antilog (loga-logb)

Calculate 6.17/1.142

a=6.17;loga=0.7903

b=1.42;logb=0.1523

0.6380

a/b=antilog (loga-logb)

a/b=antilog (0.6380)

=4.345

Calculations of powers of numbers

a^n=antilog(nloga)

Calculate $(6.75)^5$

a=6.17; loga=0.7903

n=5

0.7903x5 = 3.9515

a^n=antilog(nloga)

a^n=antilog(3.9515)

=8945

PH

Water is a weak electrolyte which dissociate only slightly to form H+ and OH- ions

H_2O----------→ H+ +OH-

The equilibrium constant for this dissociation reaction has been accurately measured, and at 250C it has value $1.8x10^{-16}$mole/litre.that is,

$$Keq = \frac{C_H + C_{OH}}{C_{H2O}}$$

The concentration of $H_2O(C_{H2O})$ in pure water may be calculated to be 1000/18 or 55.5moles/litre. Since the concentration of H_2Oin dilute aquaeous solution is essentially uncharged from that in pure H_2O, this figure may be taken as a constant. It is, in fact usually incorporated into expression for dissociation of water, to give

$$C_H x C_{OH} = 1.8x10^{-16}x55.5 = 1.01x10^{-14}$$

$$= K_w = 1.01x10^{-14}$$

At 25^0C

This new constant kw, termed as ion product of water,expresses the relation between concentration of H+ in the pure water

In 1909, Sorensen introduced the term pH as a convenient manner of expressing the concentration of H+ by means of logarithmicfunction;pH may be defined as the –ve logarithm of H^+ions

$$pH = log1/[H+] = -log[H+\}$$

$[H^+]$ denote the concentration of H^+ ions in solution.

If we now apply the term pH to the ion product expression for pure water, we obtain following equation.

$[H^+]X [OH^-] =1.0x10^{-14}$

Apply log to this equation:

$\log[H^+] + \log[OH^-] = \log(1.0x10^{-14})$

$= -14$

Multiply above equation with-1

$-\log[H^+] - \log[OH^-] =14$

We now define $- \log[OH^-]$ as pOH. We have an expression relating the pH & pOHin any aquaeous solution.

pH + pOH=14

Problems:

1. Calculate the pH of 0.001M HCl solution, assuming the complete dissociation

Solution:

Concentration of HCl=0.001M

Since HCl is completely dissociated,hence

$[H^+]= 0.001mol/dm^3$

$pH= -\log[H^+] = -\log[0.001]=3$

pH=3

2. Calculate the pH of a solution obtained by mixing 50ml of 0.2M HCl with 50ml 0.1M NaOH.

Solution:

Knowing that the product of volume in milliliters and molarity gives the number of millimoles of the acid or base, we have

Number of millimoles of the acid in the solution = 50*0.2=10

Number of millimoles of the alkali in the solution = 50*0.1=5

Number of millimoles of the acid left in the solution after the addition of alkali =10-5=5

Total volume of the solution = 50+50=100ml

Thus we have 5millimoles of the acid in 100ml of the solution or 0.05 mole of the acid per litre of the solution

Thus concentration of H^+ ions = 0.05 mol dm^{-3}.

pH of the solution = -log $[H^+]$ = -log(0.05) = 1.30

3. Calculate the pH of a solution obtained by mixing 25ml of 0.2M HCl with 50ml of 0.25M NaOH. Take $k_w=10^{-14}$ mol^2 dm^{-6}

Solution:

Knowing that the product of volume in milliliters and molarity gives the number of millimoles of the acid or the base, we have

Number of millimoles of the acid in the solution = 25*0.2=5

Number of millimoles of the alkali in the solution = 50*0.25=12.5

Number of millimoles of the alkali left in the solution after the addition of acid= 12.5-5=7.5

Total volume of the solution = 50+25=75ml

Concentration of OH^- ions = 7.5*1000/15*1000=0.10 mol dm^{-3}.

$[H^+][OH^-] = 10^{-14}$ mol^2 dm^{-6} at 25ºC

$[OH^-]$ = 0.10 mol dm^{-3}

$[H^+] = 10^{-14}$ mol^2 dm^{-6}/0.10 mol dm^{-3}=10^{-13} mol dm^{-3}

pH=-log$[H^+]$ =-log(10^{-13})=13

4. The concentration of hydrogen ion in a solution is $2.5*10^{-5}$ M. what is the solution's pH.

Solution:

$[H]^+$ = $2.5*10^{-5}$ M

Thus pH = -log $[H^+]$

$= -\log (2.5*10^{-5})$

$= -(\log 2.5+\log 10^{-5})$

$= -[0.40 + (-5)]$

$= -(0.40-5) = -(-4.6) = 4.6$

Thus the pH of the solution is 4.6.

5. Calculate the pH and pOH of a 0.0050M solution of HCl. HCl is a strong acid and completely disassociated in the solution.

$[H_3O^+] = 0.0050$ or $5.0*10^{-3}$

$pH=-\log[H^+]$

$pH=-\log[5.0*10^{-3}]$

$pH=3-\log 5.0$

$= 3- 0.70 = 2.3$

SINCE pH + pOH = 14

2.3+pOH=14

pOH=14-2.3=11.7

6. The concentration of hydrogen ion in a solution is 10^{-5}M. What is the solution's pH?

Solution: pH is the negative logarithm of 10^{-5}

$pH=-\log[H^+]==-\log[10^{-5}]=-[-5]$

pH =5

Therefore pH of the solution is 5. Therefore the solution is acidic.

In the above problem the $[H^+]$ ion concentration was stated to be $10^{-5.}$ This value can also be written as $1x10^{-5}$. The log [1] is 0. If the product rule for logarithms is used to calculate the pH for this problem, it would be equal to – [0+ (-5)], which is equal to 5.

7. What is the pH of a 0.02M solution of NaoH

   Solution: NaoH is a strong base and as such, is essentially ionized completely to Na^+ and OH^- in dilute solution. The OH^- concentration, therefore, is 0.02M, the same as the concentration of NaOH. For a strong base, the H^+ ion contribution from water is negligible and so will be ignored. The first step to solving this problem is to determine the pOH. The pOH value will then be subtracted from 14 to obtain the pH

POH=-log [0.02]

=-[-1.7]

=1.7

PH=14-1.7=12.3

Therefore the pH of the 0.02M NaOH solution is 12.3

pKa and Henderson Hesselberg equation:

According Bronsted concept of acids and bases, An acid is defined as a substance that donates a proton. A base is a substance that accepts a proton. When a bronsted acid loses a H^+ it becomes a bronsted base. The original acid is called a conjugate acid. The base created from the acid by loss of H^+ is called conjugate base.

Dissociation of an acid in the water follows formulae

$HA+H_2O \leftrightarrow H_3O^+ + A^-$

Where HA is a conjugate acid H_2O is a conjuagate base, H_3O^+ is a conjugate acid and A^- is a conjuagate base.

The acids ionization can be written as a simple dissociation as follows

$HA \leftrightarrow H^+ + A^-$

The dissociation of HA acid will occur at a certain rate characteristic of that particular acid. Notice that the reaction goes in both directions. The acid dissociates into its component ions, but the ions together again form the original acid. When the rate of dissociation into ions is equal to rate of ion ressociation, the system is said to be in equilibrium. A strong acid will reach equilibrium at the point where it is completely dissociated. A weak acid will have a lower percentage of molecules in a disossiated state and will reach equilibrium at a point less than 100% ionization. The concentration of the acid at which equilibrium occurs is called the acid-dissociation constant, designated by the symbol K_a. it is represented by the following equation.

$K_a = [H^+][A^-]/[HA]$

K_a for a weak acid is measured by its p K_a which is equivalent to negative logarthimic of K_a.

$pK_a = -\log K_a$

pH is related to p K_a by Henderson Hesselberg equation

pH= pK_a+ log [conjugate base]/[acid]

= pK_a+log[A^-]/[HA]

Henderson Hesselberg equation can thus be used to calculate the amount of acid and conjugate base to be used for the preparation of buffer.

1. What would be the pH of the solution obtained by mixing 5gms of acetic acid and 7.5gms of sodium acetate and making the volume to 500ml(dissociation constant of acetic acid at 25^{o}c is 1.75*10^{-5})

Solution:

pH= pK_a+ log [salt]/[acid]

pK_a= -log(1.75*10^{-5})=4.76

[salt]=(7.5/82*1000/500)= 0.1829 mol dm^{-3}

[acid] = (5/60*1000/500)= 1.666 mol dm^{-3}

pH= 4.76+log0.1829/0.1666=4.80

2. Calculate the pH before and after the addition of 0.01mole of NaOH to 1lit of a buffer solution that is 0.1M in acetic acid and 0.1M in sodium acetate. The dissociation constant of acetic acid is 1.75*10^{-5}.

Solution:

Prior to the addition of NaOH,

[CH_3COOH] = 0.1M; [CH_3COO^-] = 0.1M

pH= pK_a + log [CH_3COO^-]/[CH_3COOH]

pK_a=-logK_a=-log(1.75*10^{-5})=4.76

pH=4.76+log0.1/0.1=4.76+log1=4.76+0=4.76

After the addition of 0.01mole of NaOH, some of the acetic acid is neutralized so that the concentration of the weak acid is diminished while that of the salt(acetate ion) is increased. Thus, we have

$[CH_3COOH]=0.1-0.01=0.09$ mol dm^{-3}

$[CH_3COO^-]=0.1+0.01=0.11$ mol dm^{-3}

Hence, the pH of the buffer is given by

pH=4.76+log0.11/0.09=4.76+0.087=4.847

Pk_b

If a buffer solution consists of amixture of weak base and its salt then its pOH is given by

$[OH^-] = K_b$ [BASE]/[SALT]

$pOH=pK_b+\log$ [salt]/[base]

The concentration of the base at which equilibrium occurs is called the base-dissociation constant, designated by the symbol K_b.

Knowing Poh, the pH Can easily be calculated from the well known relationship

$pH+ pK_b =pK_w=14$

1. A buffer solution contains 0.2mole of NH4OH and 0.25mole of NH_4Cl per litre. Calculate the pH of the solution. Dissociation constant of NH_4OH at room temperature is $1.81*10^{-5}$.

Solution:

$pOH=pK_b+\log$ [salt]/[base]

$pK_b= -\log K_b=-\log(1.81*10^{-5})=4.7423$

pOH=4.7423+log0.25/0.20=4.7423+(log0.25-log0.20)=4.839

pH=14-4.839=9.161

What are the (a) H^+ ion concentration (b)p^H & (c) OH^- ion concentration &

(d)p^{OH} of a 0.004M Solution of HCl?

(a) HCl being a strong inorganic acid it is 100% ionized in dilute solution. Hence whwn 0.004M of HCl is introduced with 1 litre of H2O, it immediately dissociate with 0.004M H^+ & 0.004M of Cl^-

(b) p^H =-log[H^+] (where [H^+]= 0.004M=$4x10^{-4}$M)

= -log4x10-3 =log1/4x10^{-3}

=log0.25x10^3

= log 25x10^1

=log5^2+log10 =2log5+log10

=2x0.699+1

=2.398

(c)[H^+] [OH^-]=K_w=1x10^{-14}

[OH^-]=1x10^{-14}/[H^+]=1x10^{-14}/4x10^{-3}=0.25x10^{-11}

[OH^-]=0.25x10^{-11}=25x10^{-13}

(a) pOH=-log[OH^-]

=-log[25x10^{-13}]=- log(1/4x10^{-11})=log4/10^{-11}

=log4-log10^{-11}= log2^2+11log10 = 2log2+11log10

2x0.3010+11

pOH=0.6020+11= 11.6020